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LINEAR SYSTEMS

Incomplete Measurements-Based Exponential Stabilization and Asymptotic Estimation of Solutions of Linear Neutral Systems

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Abstract—This paper is devoted to a linear autonomous differential-difference system of neutral type with lumped delays. For such systems, we propose existence criteria for output-feedback controllers based on incomplete measurements that ensure a given spectrum of the closed-loop system or its exponential stabilization. In addition, we prove existence criteria for observers forming asymptotic estimates with errors described by linear homogeneous systems with a predetermined characteristic quasipolynomial or exponential stability. All the considerations are constructive and contain a method for designing a corresponding controller or observer.

Keywords: differential-difference system, neutral type, delay, exponential stabilization, modal controllability, controller, observer

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1. INTRODUCTION

The delay effect is inherent in almost all control processes. Therefore, it should be taken into account when building engineering, economic, and other models [1–4]. The general theory of delayed systems, as well as their applications, was studied in rather many works (for example, see the introduction in [3, 4]). In this paper, we investigate the stabilization problem for neutral delay systems. Such systems describe the behavior of plants and processes whose rate of evolution depends on both their previous states and their velocities, e.g., the motion of a pendulum with a viscous filler [2], the plunge grinding model, and plants whose dynamics are described by systems with distributed delays (in particular, telegraph equations). Let us provide other particular examples of stabilization problems for linear systems of neutral type. When studying the oscillations of the current collector of a moving locomotive far from the support (placed behind the current collector), it is necessary to consider the effect of the reflected waves of the contact wire from the strings supporting this wire and from the support placed in front of the moving current collector. For such a mechanical system, one naturally encounters the stabilization problem [5]. Another example is the stabilization problem of a system arising during the translational and rectilinear motion of some mass under the action of a linear restoring force proportional to the coordinate and some nonconservative force [6, p. 235]. Some time is needed to trigger the system's sensitive elements detecting the displacement, velocity, and acceleration of the mass, as well as the relay and servomotor; therefore, one obtains a model in the form of a linear autonomous system of neutral type [6, p. 235].

Research into the stabilization problem of delayed systems was initiated in [7, 8] and then picked up by many scientists [9–16] (see also the bibliography therein). However, despite a rather large flow of publications in this direction, the stabilization problem has not been fully studied to date.

In general, the spectrum of linear systems with aftereffect is infinite, so the analysis and subsequent elimination of unstable eigenvalues from the spectrum requires some computational effort [15]. In this regard, a more universal approach to stabilize the system is to assign a finite spectrum, usually consisting of numbers with negative real parts [17–19]. A significant disadvantage of this system stabilization approach is the solvability conditions of the corresponding (spectrum assignment) problem, which are more stringent compared to the stabilization conditions.

Modal controllability is a more general problem than finite spectrum assignment: it is required to tune the coefficients of the characteristic quasipolynomial of a system [20–22].

The Lyapunov–Krasovskii and Lyapunov–Razumikhin methods are effective for analyzing the stability of delayed systems. They allow formulating the solvability conditions of the control problem in terms of matrix inequalities [23, Chap. 3–7]. This approach to controller analysis and design provides constructive finite-dimensional conditions for its existence and can be extended to other problems. For example, the control law designed in [24] limits the influence of disturbances and measurement noise; the stability conditions of input data presented therein were described in terms of matrix inequalities.

In contrast to the above method, based to a greater extent on the differential properties of the control system, the approach proposed in this article is of a purely algebraic nature. A polynomial $\det W(p,\lambda)$, where $W(p,e^{-ph})$ is the characteristic matrix of a closed-loop system (in the case of a dynamic controller), is treated as an element of an ideal $\mathfrak{I}$ generated by a system of polynomials, i.e., algebraic complements to the elements of the last row of the matrix $W(p,\lambda)$. Therefore, the class of possible characteristic quasipolynomials $\det W(p,e^{-ph})$ can be described by computing the Gröbner basis of the ideal $\mathfrak{I}$. This circumstance reduces all controller/observer design computations to operations in the ring of polynomials. This idea was utilized in [19, 25, 26] to construct a feedback controller ensuring, after a finite time, zero values for all components of the original open-loop system, i.e., a finite stabilization controller [27]. (In other words, such a controller completely damps/calms the original open-loop system.) Such a problem is solved by constructing appropriate feedback so that the closed-loop system becomes a finite-spectrum system pointwise degenerate in the directions corresponding to the components of the solution vector of the original system [19, 25]. These ideas were extended to systems of neutral type in [26] and systematized in the monograph [4]. The next step in exploring the finite stabilization problem was the development of output-feedback controllers based on available output observations. For delayed single-input single-output (SISO) systems, such a problem was considered in [27]; for multi-input systems of neutral type, in [28, 29].

In this paper, utilizing the spectrum control methods for neutral systems [15] and the block diagrams of feedback controllers with incomplete measurements [28, 29], we prove existence criteria for output-feedback controllers based on available output observations that solve the problems of modal controllability and stabilization. In addition, we propose methods for designing two types of asymptotic observers and establish criteria for their existence.

2. NOTATION

Consider a linear autonomous differential-difference system of neutral type with commensurate delays:

$$\dot{x}(t) = A_0 x(t) + \sum_{j=1}^{m} \left(A_j x(t-jh) + D_j \dot{x}(t-jh) \right) + \sum_{j=0}^{m} b_j u(t-jh), \quad t > 0, \tag{1}$$

$$y(t) = \sum_{j=0}^{m} c_j' x(t-jh), \quad t \geqslant 0, \tag{2}$$

$$x(t) = \eta(t), \quad t \in [-mh, 0]. \tag{3}$$

Here, $x \in \mathbb{R}^n$ is the column vector of the solution of system (1) ($n \geqslant 2$); $0 < h$ is a constant delay; $A_0, A_j, D_j \in \mathbb{R}^{n\times n}$ and $b_j \in \mathbb{R}^n$, $c_j' \in \mathbb{R}^n$; the dash symbol ($'$) indicates the transpose; u is the control input (a scalar piecewise continuous function); y is the observed output (a scalar signal). By assumption, the initial function η is continuous with a piecewise continuous derivative. In this case, there exists a unique continuous solution with a piecewise continuous derivative. Throughout this paper, the initial function η is supposed to be unknown.

This study pursues the following objective: based on available observations of the output of (2), design output-feedback controllers that ensure a given characteristic quasipolynomial of the closed-loop system or its exponential stabilization. The remainder of this paper is organized as follows. First (see Section 3), two types of asymptotic observers are constructed using the controller design methods from [15]. Then (Section 4), in order to obtain feedback controllers based on available output observations, additional loops are incorporated into the controller structure from [15] in the form of asymptotic observers according to the principle developed in [28, 29]. Finally, an illustrative example is given in Section 5.

Let p, $\lambda \in \mathbb{C}$, where $\mathbb{C}$ is the set of complex numbers. Also, we introduce the following notation:

$$A(p,\lambda) = A_0 + \sum_{j=1}^{m}(A_j + pD_j)\lambda^j; \tag{4}$$

$W(p, e^{-ph}) = pI_n - A(p, e^{-ph})$ is the characteristic matrix ($I_n \in \mathbb{R}^{n\times n}$ means an identity matrix of order n); $w(p, e^{-ph}) = |W(p, e^{-ph})|$ is the characteristic quasipolynomial of the homogeneous ($u = 0$) system (1). From this point onwards, $|W|$ means the determinant of an arbitrary square matrix W.

Let $\phi \in \mathbb{N}$ be an arbitrary number. A quasipolynomial $d(p, e^{-ph})$, where

$$d(p,\lambda) = \sum_{i=0}^{\phi} \theta_i(\lambda)p^i$$

and $\theta_i(\lambda)$ are some polynomials with $\theta_\phi(0) = 1$, will be called a quasipolynomial of neutral type. If $\theta_\phi(\lambda) = 1$, we have a quasipolynomial $d(p, e^{-ph})$ of delayed type as a special case. The characteristic quasipolynomial $w(p, e^{-ph})$ of the homogeneous system (1) is in general a quasipolynomial of neutral type and $\deg_p w(p,\lambda) = n$.

Let $\mathbb{R}^{r\times m}[\lambda]$ and $\mathbb{C}^{r\times m}[\lambda]$ be the sets of matrices of dimensions $r \times m$ whose elements are polynomials of the variable λ with real and complex coefficients, respectively. (If $r = m = 1$, the superscript will be omitted.) In addition, let λ_h and p_D be the shift and differentiation operators, respectively, i.e., $p_D^i \lambda_h^j f(t) = f^{(i)}(t - jh)$ for a function f and integers $i, j \geqslant 0$.

To write the equations of the controllers and observers compactly, we introduce the set $\mathfrak{Q}^{r\times m}$ $\left(\mathfrak{Q}^{1\times 1} = \mathfrak{Q}\right)$, consisting of all mappings $\mathcal{Q} : f \mapsto \mathcal{Q}[f]$, where $f(t)$, $t \in \mathbb{R}$ is an arbitrary continuous (scalar or vector) function with a piecewise continuous derivative. (Square brackets are used to distinguish mappings and functions.) Each mapping $\mathcal{Q} \in \mathfrak{Q}^{r\times m}$ is given by the following elements: 1) $q_i(\lambda) \in \mathbb{R}^{r\times m}[\lambda]$, $i = 0, 1$; 2) $P = \{\alpha_k \pm \mathbf{i}\beta_k,\ \alpha_k,\ \beta_k \in \mathbb{R},\ k = \overline{1, n_1}\}$, representing the set of real and complex conjugate numbers ($\mathbf{i}$ denotes the imaginary unit); 3) $\widehat{q}_{ki}(\lambda) \in \mathbb{C}^{r\times m}[\lambda]$, $k = \overline{1, n_1}$, $i = \overline{1, n_2}$ ($n_1 \geqslant 1$, $n_2 \geqslant 0$ are integers). Each mapping of this kind acts according to the rule

$$\mathcal{Q}[f(t)] = q_0(\lambda_h)f(t) + q_1(\lambda_h)\dot{f}(t-h) + \sum_{k=1}^{n_1}\sum_{i=0}^{n_2}\int_0^h \widehat{q}_{ki}(\lambda_h)f(t-s)e^{p_k s}\frac{s^i}{i!}\,ds, \quad t > 0, \tag{5}$$

where $p_k \in P$. The matrices $\widehat{q}_{ki}(\lambda)$ in (5) and the set P possess the following property: with the Euler formula applied ($e^{\mathbf{i}\varphi} = \cos\varphi + \mathbf{i}\sin\varphi$), the expression (5) becomes

$$\mathcal{Q}[f(t)] = q_0(\lambda_h)f(t) + q_1(\lambda_h)\dot{f}(t-h) + \sum_{j=0}^{\widehat{n}_1}\int_0^h r_j(s)f(t-jh-s)\,ds, \tag{6}$$

where $\widehat{n}_1 = \max_{k,i}\{\deg_\lambda \widehat{q}_{ki}(\lambda)\}$, $r_j(s) = \sum_{k=1}^{n_1} e^{\alpha_k s}(\cos(\beta_k s)\nu_{jk}(s) + \sin(\beta_k s)\mu_{jk}(s))$, $(\alpha_k + \mathbf{i}\beta_k) \in P$, and $\nu_{jk}(s), \mu_{jk}(s) \in \mathbb{R}^{r\times m}[s]$ ($\deg_s \nu_{kj} \leqslant n_2$, $\deg_s \nu_{kj} \leqslant n_2$). Thus, all expressions in the relation (6) are real numbers.

When the original system is closed by controllers containing the terms (5) (equivalently, the terms (6)), distributed delays described by integral terms in (6) may appear in the closed-loop system. In this case, the distributed delay terms (see the expression (5)) are associated with the expressions $\widehat{q}_{ki}(e^{-ph})\int_0^h e^{-(p-p_k)s}s^i/i!\,ds$ in the characteristic matrix of the closed-loop system. Calculating the integrals of these expressions and then letting $\lambda = e^{-ph}$ yield the integer fractional rational functions [19]

$$\int_0^h e^{-(p-p_k)s}s^i/i!ds\bigg|_{e^{-ph}=\lambda} = \frac{(-1)^{i+1}}{i!}\frac{d^i}{dp^i}\left(\frac{\lambda - e^{-p_k h}}{e^{-p_k h}(p-p_k)}\right), \quad i = 0, 1, \ldots. \tag{7}$$

The expression (5) (or (6)) is associated with the matrix

$$\mathcal{Q}[e^{pt}]e^{-pt}\bigg|_{e^{-ph}=\lambda} = Q(p,\lambda)$$

in the characteristic matrix of the closed-loop system, where

$$Q(p,\lambda) = q_0(\lambda) + p\lambda q_1(\lambda) + q(p,\lambda) \tag{8}$$

and $q(p,\lambda)$ is a matrix of fractional rational functions of the form $\frac{\omega_1(p,\lambda)}{\omega_2(p)}$, proper in the variable p ($\omega_1(p,\lambda)$ and $\omega_2(p)$ are polynomials with complex coefficients such that $\deg_p \omega_1(p,\lambda) < \deg_p \omega_2(p)$). We further suppose that if $\widehat{W}(p, e^{-ph})$ is the characteristic matrix of a neutral system with a distributed delay given by (6), then the matrix $\widehat{W}(p,\lambda)$ is obtained by first calculating the integrals (7) and then letting $e^{-ph} = \lambda$ in the resulting expression.

For a given mapping $\mathcal{Q}$ (5), the transposed mapping $\mathcal{Q}'$ is the one obtained from (5) by replacing $q_0(\lambda)$, $q_1(\lambda)$, and $\widehat{q}_{ki}(\lambda)$ with $q_0'(\lambda)$, $q_1'(\lambda)$, and $\widehat{q}_{ki}'(\lambda)$, respectively.

3. ASYMPTOTIC ESTIMATION OF THE SOLUTION

In this section, we construct observers forming asymptotic estimates of the solution of the original system (2) from the measurements (1) with errors vanishing at a given or exponential rate, determined by the roots of the characteristic quasipolynomial. Further, these results will be needed to design a stabilizing output-feedback controller based on available output observations.

We define the following linear system of neutral type:

$$\begin{aligned}\dot{z}_1(t) &= A(p_{_D},\lambda_h)z_1(t) + \mathcal{L}_1[z_2(t)] + b(\lambda_h)u(t),\\ \dot{z}_2(t) &= \beta_0(p_{_D})c'(\lambda_h)z_1(t) + \mathcal{L}_2[z_2(t)] - \beta_0(p_{_D})y(t), \quad t > 0,\end{aligned} \tag{9}$$

where the matrix $A(p,\lambda)$ is given by (4), $\mathcal{L}_1 \in \mathfrak{Q}^{n\times 1}$, $\mathcal{L}_2 \in \mathfrak{Q}^{1\times 1}$, $\beta_0(p) \in \mathbb{R}_0[p]$, and $\mathbb{R}_0[p] = \{1,\, p + \widehat{\alpha} : \widehat{\alpha} \in \mathbb{R}\}$ is the set of polynomials that have the form $p + \widehat{\alpha}$ or are equal to 1. For system (9), we choose any initial condition

$$z(t) = \varphi(t), \quad t \in [-h_0, 0], \tag{10}$$

where φ is a continuous function with a piecewise continuous derivative and h_0 is the delay of system (9).

We take the component z_1 of the solution vector $z = \mathrm{col}[z_1, z_2]$ of system (9) as an estimate of the solution x of system (1), (2) given the control input u. Obviously, the function $\zeta = z_1 - x$, representing the error of the estimate z_1 of the solution x, is a component of the solution of the homogeneous system

$$\begin{gathered}\dot{\zeta}(t) = A(p_D, \lambda_h)\zeta(t) + \mathcal{L}_1[z_2(t)], \\ \dot{z}_2(t) = \beta_0(p_D)c'(\lambda_h)\zeta(t) + \mathcal{L}_2[z_2(t)], \quad t > 0.\end{gathered} \tag{11}$$

Consider the characteristic matrix $W_z(p, \lambda)$ of system (11) (the homogeneous ($u = 0$) system (9)):

$$W_z(p, \lambda) = \begin{bmatrix} pI_n - A(p, \lambda) & -L_1(p, \lambda) \\ -\beta_0(p)c'(\lambda) & p - L_2(p, \lambda) \end{bmatrix}, \tag{12}$$

where $L_i(p, \lambda) = \mathcal{L}_i[e^{pt}]e^{-pt}$. Let us introduce the polynomial

$$g(p, \lambda) = \sum_{i=0}^{n+1} p^i g_i(\lambda), \quad g_i(\lambda) \in \mathbb{R}[\lambda], \quad g_{n+1}(0) = 1. \tag{13}$$

Generally speaking, the quasipolynomial $d(p, e^{-ph})$ is of neutral type.

Definition 1. System (1), (2) is said to have an observer (9) with a given characteristic polynomial if, for any polynomial (13), there exist $\mathcal{L}_1 \in \mathfrak{Q}^{n\times 1}$, $\mathcal{L}_2 \in \mathfrak{Q}^{1\times 1}$, and $\beta_0(p) \in \mathbb{R}_0[\lambda]$ such that

$$\left|W_z(p, \lambda)\right| = g(p, \lambda). \tag{14}$$

Remark 1. The main goal of observer design is to obtain an estimate for the solution of an original system. Therefore, when designing an observer with a given characteristic quasipolynomial, the quasipolynomial (13) should be chosen so that system (11) be asymptotically or exponentially stable. Regarding the computational complexity of solving system (11), the most convenient choice is a polynomial (13) that does not depend on the variable λ and has roots with negative real parts.

Definition 2. System (1), (2) is said to have an exponentially stable observer (9) if there exist $\mathcal{L}_1 \in \mathfrak{Q}^{n\times 1}$, $\mathcal{L}_2 \in \mathfrak{Q}^{1\times 1}$, and $\beta_0(p) \in \mathbb{R}_0[p]$ such that system (11) is exponentially stable.

Remark 2. A linear homogeneous autonomous system of neutral type is exponentially stable if and only if [14] its characteristic quasipolynomial possesses exponential stability (i.e., the roots p_i of the characteristic equation satisfy the inequality $\mathbf{Re}\, p_i < \varepsilon\ \exists \varepsilon < 0$). In this case, the difference equation describing the jump behavior of the first derivatives of the solution is exponentially stable [14]. We illustrate the above on an example of the system

$$\dot{x}(t) = \mathcal{Q}[x(t)], \tag{15}$$

where the mapping $\mathcal{Q}$ is given by (6). (All matrices in (6) have dimensions $n \times n$.) Let $W_0(p, e^{-ph})$ be the characteristic matrix of system (15), $W_0(p, \lambda) = p(I_n - \lambda q_1(\lambda)) - q_0(\lambda) - q(p, \lambda)$ (see (8)). We introduce the sets

$$\Delta_0 = \left\{p \in \mathbb{C} : \left|W_0(p, e^{-ph})\right| = 0\right\}, \quad \Delta_1 = \left\{\lambda \in \mathbb{C} : \left|I_n - \lambda q_1(\lambda)\right| = 0\right\}. \tag{16}$$

For the exponential stability of system (15), it is necessary and sufficient that

$$\mathbf{Re}\, p < -\varepsilon \quad \exists \varepsilon > 0, \quad p \in \Delta_0. \tag{17}$$

In this case, the exponential stability of the difference equation implies

$$|\lambda| > 1, \quad \lambda \in \Delta_1. \tag{18}$$

Consider system (1). Denoting $D(\lambda) = \sum_{j=1}^{m} \lambda^j D_j$, we formulate existence criteria for an observer with a given characteristic quasipolynomial.

Theorem 1. *For system* (1), (2) *to have an observer* (9) *with a given characteristic quasipolynomial, it is necessary and sufficient that*

$$\operatorname{rank}\begin{bmatrix} W(p, e^{-ph}) \\ c'(e^{-ph}) \end{bmatrix} = n \quad \forall p \in \mathbb{C}, \quad \operatorname{rank}\begin{bmatrix} I_n - D(\lambda) \\ c'(\lambda) \end{bmatrix} = n \quad \forall \lambda \in \mathbb{C}. \tag{19}$$

The proof of this result is provided in the Appendix.

The following theorem represents an existence criterion for an exponentially stable observer.

Theorem 2. *For system* (1), (2) *to have an exponentially stable observer* (9), *it is necessary and sufficient that*

$$\begin{gathered} \operatorname{rank}\begin{bmatrix} W(p, e^{-ph}) \\ c'(e^{-ph}) \end{bmatrix} = n \quad \forall p \in \mathbb{C}, \quad \mathbf{Re}\, p \geqslant \varepsilon_1, \quad \exists \varepsilon_1 < 0, \\ \operatorname{rank}\begin{bmatrix} I_n - D(\lambda) \\ c'(\lambda) \end{bmatrix} = n \quad \forall \lambda \in \mathbb{C}, \quad |\lambda| \leqslant 1. \end{gathered} \tag{20}$$

See the proof in the Appendix.

4. MODAL CONTROLLABILITY AND EXPONENTIAL STABILIZATION

We define a dynamic output-feedback controller based on available output measurements:

$$\begin{aligned} u(t) &= \alpha_0(p_D)x_1(t), \\ \dot{x}_1(t) &= \mathcal{Q}_{11}[x_1(t)] + \mathcal{Q}_{12}[x_2(t)], \\ \dot{x}_2(t) &= b(\lambda_h)\alpha_0(p_D)x_1(t) + A(p_D, \lambda_h)x_2(t) + \mathcal{Q}_{23}[x_3(t)], \\ \dot{x}_3(t) &= \alpha_1(p_D)c'(\lambda_h)x_2(t) + \mathcal{Q}_{33}[x_3(t)] - \alpha_1(p_D)y(t), \quad t > 0, \end{aligned} \tag{21}$$

where $x_1,\, x_3 \in \mathbb{R}$ and $x_2 \in \mathbb{R}^n$ are auxiliary variables; $\mathcal{Q}_{11} \in \mathfrak{Q}$, $\mathcal{Q}_{12} \in \mathfrak{Q}^{1\times n}$, $\mathcal{Q}_{23} \in \mathfrak{Q}^{n\times 1}$, $\mathcal{Q}_{33} \in \mathfrak{Q}$, and $\alpha_i(p) \in \mathbb{R}_0[p]$, $i = 0, 1$.

Let us close system (1), (2) with the controller (21). Obviously, system (1), (2), (21) is linear inhomogeneous autonomous of neutral type with commensurable lumped and distributed delays, and its inhomogeneous part depends on the output $y(t)$. Following (2), we replace the function $y(t)$ in the inhomogeneous part with $c'(\lambda_h)x(t)$ to obtain the homogeneous one. The characteristic matrix $\overline{W}(p, \lambda)$ of this homogeneous system is given by

$$\overline{W}(p,\lambda) = \begin{bmatrix} pI_n - A(p,\lambda) & -\alpha_0(p)b(\lambda) & 0_{n\times n} & 0_{n\times 1} \\ 0_{1\times n} & p - Q_{11}(p,\lambda) & -Q_{12}(p,\lambda) & 0 \\ 0_{n\times n} & -\alpha_0(p)b(\lambda) & pI_n - A(p,\lambda) & -Q_{23}(p,\lambda) \\ \alpha_1(p)c'(\lambda) & 0 & -\alpha_1(p)c'(\lambda) & p - Q_{33}(p,\lambda) \end{bmatrix}, \tag{22}$$

where $Q_{ij}(p, \lambda) = \mathcal{Q}_{ij}[e^{pt}]e^{-pt}$ and $0_{i\times j} \in \mathbb{R}^{i\times j}$ $(i,\, j > 1)$ is a zero matrix of appropriate dimensions.

Definition 3. System (1), (2) is said to be modally controllable (by the output) if, for any polynomial

$$\chi(p, \lambda) = \chi_1(p, \lambda)\chi_2(p, \lambda), \tag{23}$$

where $\chi_k(p, \lambda) = \sum_{i=0}^{n+1} p^i \chi_{ki}(\lambda)$, $\chi_{ki} \in \mathbb{R}[\lambda]$, $k = 1, 2$, and $\chi_{k\,n+1}(0) = 1$, there exists a controller (21) such that the characteristic matrix of the closed-loop system (1), (2), (21) satisfies

$$\left|\overline{W}(p, \lambda)\right| = \chi(p, \lambda). \tag{24}$$

Generally speaking, the quasipolynomial $\chi(p, e^{-ph})$ is of neutral type.

Definition 4. System (1), (2) is said to be exponentially stabilizable (by the output) if there exists a controller (21) such that the closed-loop system (1), (2), (21) is exponentially stable.

The following theorems are criteria for the modal controllability and exponential stabilizability of system (1), (2) in the class of controllers (21).

Theorem 3. *System* (1), (2) *is modally controllable in the class of controllers* (21) *if and only if*

$$\begin{gathered}\operatorname{rank}\big[W(p,e^{-ph}),b(e^{-ph})\big]=n\quad\forall p\in\mathbb{C},\\ \operatorname{rank}\big[I_n-D(\lambda),\,b(\lambda)\big]=n\quad\forall\lambda\in\mathbb{C},\end{gathered}\tag{25}$$

and conditions (19) *hold.*

See the proof in the Appendix.

Theorem 4. *System* (1), (2) *is exponentially stabilizable in the class of controllers* (21) *if and only if*

$$\begin{gathered}\operatorname{rank}\big[W(p,e^{-ph}),b(e^{-ph})\big]=n\quad\forall p\in\mathbb{C},\quad \mathbf{Re}\,p\geqslant\varepsilon_0,\quad\exists\varepsilon_0<0,\\ \operatorname{rank}\big[I_n-D(\lambda),\,b(\lambda)\big]=n\quad\forall\lambda\in\mathbb{C},\quad|\lambda|\leqslant 1,\end{gathered}\tag{26}$$

and conditions (20) *hold.*

See the proof in the Appendix.

5. EXAMPLE

Let system (1), (2) be of the second order and be described by the following matrices and delay:

$$\begin{gathered}A(p,\lambda)=\begin{bmatrix}-\dfrac{1}{2}p\lambda & -3+\lambda\\ -\dfrac{1}{3} & -\dfrac{5}{12}\lambda\end{bmatrix},\quad b(\lambda)=\begin{bmatrix}0\\ 2\lambda-\lambda^2\end{bmatrix},\\ c(\lambda)=[0,\ -1],\quad h=\ln 2.\end{gathered}\tag{27}$$

The original system with the matrices (27) has an infinite spectrum, and its characteristic quasipolynomial ($\lambda=e^{-ph}$) is given by

$$w(p,\lambda)=\frac{1}{2}p^2(\lambda+2)+\frac{5}{24}p\lambda(\lambda+2)+\frac{\lambda}{3}-1.$$

The quasipolynomial $w(p,e^{-ph})$ has a positive root since $w(0,1)=-\frac{2}{3}<0$; $\lim\limits_{p\to+\infty}w(p,e^{-ph})=+\infty$. Thus, the unperturbed system is not exponentially stable.

Obviously, the first condition in (25) is violated for $p=-1$ and the second for $\lambda=-1$. This means the validity of conditions (26). The first condition in (19) also holds but the second condition fails for $\lambda=-2$; therefore, conditions (20) are true. Thus, the results of [28] (the design of an incomplete measurements-based controller ensuring complete stabilization (simultaneously finite and asymptotic stabilization and finite spectrum assignment) or those of [29] (only finite stabilization) are inapplicable here. However, the conditions of Theorem 4 are satisfied, so we can construct a controller based on incomplete measurements to exponentially stabilize the closed-loop system. Looking ahead, note that the set of roots of the characteristic quasipolynomial of this closed-loop system contains the points $p=-1$ and the roots of the equation $e^{-ph}=\lambda$ with $\lambda=-2$, at which conditions (19), (25) are violated.

Now we proceed to the controller design (21).

1. Following [15], we construct a controller (A.5). Necessary calculations [15] yield

$$
\begin{aligned}
u(t) &= x_1(t),\\
\dot{x}_1(t) &= \left[\frac{5}{6} - \frac{1}{6}\lambda_h\right]\dot{x}_1(t-h) + \left[-6 + \frac{65}{12}\lambda_h - \frac{29}{24}\lambda_h^2\right]x_1(t)\\
&+ \int_0^h (-12 + 6\lambda_h)x_1(t-s)e^s ds + \left[\frac{-5}{72}, \frac{5}{72}\right]\dot{x}(t-h)\\
&+ \left[\frac{-223}{72} - 2\lambda_h, \frac{25}{3} + \frac{185}{288}\lambda_h\right]x(t) + \int_0^h \left[1, -\frac{9}{2}\right]e^s x(t-s)ds.
\end{aligned}
\tag{28}
$$

In this case, the matrix (A.6) has the form

$$
W_x(p,\lambda) = \begin{bmatrix} p + \dfrac{p\lambda}{2} & 3-\lambda & 0\\ \dfrac{1}{3} & p + \dfrac{5}{12}\lambda & (2-\lambda)\lambda\\ \nu_1(p,\lambda) & \nu_2(p,\lambda) & \nu_3(p,\lambda)\end{bmatrix}, \tag{29}
$$

where

$$
\begin{aligned}
\nu_1(p,\lambda) &= \frac{5p\lambda}{72} - \frac{1-2\lambda}{p-1} + \frac{223}{72} + 2\lambda, \quad \nu_2(p,\lambda) = -\frac{5p\lambda}{72} + \frac{9(1-2\lambda)}{2(p-1)} - \frac{25}{3} - \frac{185\lambda}{288},\\
\nu_3(p,\lambda) &= p - \frac{5p\lambda}{6} + \frac{p\lambda^2}{6} + 6\frac{(1-2\lambda)(2-\lambda)}{p-1} + 6 - \frac{65\lambda}{12} + \frac{29\lambda^2}{24}.
\end{aligned}
$$

Straightforward calculations finally lead to $|W_x(p,\lambda)| = (1-\frac{\lambda}{3})(1-\frac{\lambda}{2})(1+\frac{\lambda}{2})(p+1)(p+2)(p+3)$.

2. We construct an exponentially stable observer (9). For this purpose, following the proof of Theorem 4, we construct a controller (A.2) for system (A.1) that exponentially stabilizes the closed-loop system (9), (A.2). Then, according to (A.3), we obtain an exponentially stable observer (9) with

$$
\begin{aligned}
\mathcal{L}_1[z_2] &= \begin{bmatrix} \dfrac{-79}{4}\lambda_h - \dfrac{31}{24}\lambda_h^2 - \dfrac{5}{24}\lambda_h^3 - 36\\ \dfrac{25}{288}\lambda_h^3 - \dfrac{155}{144}\lambda_h^2 + \dfrac{8}{3}\lambda_h + 12\end{bmatrix} z_2(t),\\
\mathcal{L}_2[z_2] &= \frac{-1}{2}\dot{z}_2(t-h) + \left(\frac{5}{24}\lambda_h^2 - \frac{31}{12}\lambda_h - 6\right)z_2(t).
\end{aligned}
$$

The characteristic matrix (12) has the form

$$
W_z(p,\lambda) = \begin{bmatrix} \dfrac{p(2+\lambda)}{2} & 3-\lambda & \dfrac{79}{4}\lambda + \dfrac{31}{24}\lambda^2 + \dfrac{5}{24}\lambda^3 + 36\\ \dfrac{1}{3} & \dfrac{5\lambda}{12} + p & -\dfrac{25}{288}\lambda^3 + \dfrac{155}{144}\lambda^2 - \dfrac{8}{3}\lambda - 12\\ 0 & 1 & p + \dfrac{1}{2}p\lambda - \dfrac{5}{24}\lambda^2 + \dfrac{31}{12}\lambda + 6\end{bmatrix} \tag{30}
$$

and $|W_z(p,\lambda)| = \left(1 + \frac{1}{2}\lambda\right)^2 (p+3)(p+2)(p+1)$.

3. Using the parameters of the above controller and observer, we construct a controller (21):

$$
\begin{aligned}
&u(t) = x_1(t),\\
&\dot{x}_1(t) = \left[\frac{5}{6} - \frac{1}{6}\lambda_h\right]\dot{x}_1(t-h) + \left[-6 + \frac{65}{12}\lambda_h - \frac{29}{24}\lambda_h^2\right]x_1(t)\\
&\quad + \int_0^h (-12 + 6\lambda_h)x_1(t-s)e^s ds + \left[\frac{-5}{72}, \frac{5}{72}\right]\dot{x}_2(t-h)\\
&\quad + \left[\frac{-223}{72} - 2\lambda_h, \frac{25}{3} + \frac{185}{288}\lambda_h\right]x_2(t) + \int_0^h \left[1, -\frac{9}{2}\right]e^s x(t-s)ds,\\
&\dot{x}_2(t) = \begin{bmatrix} 0 \\ 2\lambda_h - 2\lambda_h^2 \end{bmatrix} x_1(t) + \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{bmatrix}\dot{x}_2(t-h) + \begin{bmatrix} 0 & -3+\lambda_h \\ -\frac{1}{3} & -\frac{5}{12}\lambda_h \end{bmatrix} x_2(t)\\
&\quad + \begin{bmatrix} \frac{-79}{4}\lambda_h - \frac{31}{24}\lambda_h^2 - \frac{5}{24}\lambda_h^3 - 36 \\ \frac{25}{288}\lambda_h^3 - \frac{155}{144}\lambda_h^2 + \frac{8}{3}\lambda_h + 12 \end{bmatrix} x_3(t),\\
&\dot{x}_3(t) = \frac{-1}{2}\dot{x}_3(t-h) + \left(\frac{5}{24}\lambda_h^2 - \frac{31}{12}\lambda_h - 6\right)x_3(t) + ([0,\ 1]x_2(t) - y(t)).
\end{aligned}
\tag{31}
$$

The matrix of the closed-loop system (22) has the form

$$
\overline{W}(p,\lambda) = \begin{bmatrix}
\frac{p(2+\lambda)}{2} & 3-\lambda & 0 & 0 & 0 & 0\\
\frac{1}{3} & \frac{5\lambda}{12}+p & (2-\lambda)\lambda & 0 & 0 & 0\\
0 & 0 & \nu_3(p,\lambda) & \nu_1(p,\lambda) & \nu_2(p,\lambda) & 0\\
0 & 0 & 0 & \frac{p(2+\lambda)}{2} & 3-\lambda & \frac{79}{4}\lambda + \frac{31}{24}\lambda^2 + \frac{5}{24}\lambda^3 + 36\\
0 & 0 & (2-\lambda)\lambda & \frac{1}{3} & \frac{5\lambda}{12}+p & -\frac{25}{288}\lambda^3 + \frac{155}{144}\lambda^2 - \frac{8}{3}\lambda - 12\\
0 & -1 & 0 & 0 & 1 & p + \frac{1}{2}p\lambda - \frac{5}{24}\lambda^2 + \frac{31}{12}\lambda + 6
\end{bmatrix}.
$$

Straightforward calculations yield

$$
|\overline{W}(p,\lambda)| = \left(1 - \frac{1}{2}\lambda\right)\left(1 - \frac{1}{3}\lambda\right)\left(1 + \frac{1}{2}\lambda\right)^3 (p+3)^2(p+2)^2(p+1)^2.
$$

6. CONCLUSIONS

This paper has been devoted to linear autonomous differential-difference systems of neutral type with a scalar control input and an observable output. For such systems, we have derived modal controllability and exponential stabilizability criteria in the class of output-feedback controllers (function of the observed output). Modal controllability provides wider system design capabilities compared to stabilizability. In particular, it is possible to specify the rate of convergence to zero (vanishing) for the system solution by tuning the coefficients of the characteristic quasipolynomial.

Alternatively, it is possible to ensure a finite spectrum, making the system simpler from a (subsequent) control standpoint. However, the requirements for the system parameters imposed by the modal controllability criterion are more stringent than the conditions of exponential stabilizability.

Two types of asymptotic observers have been developed, namely, an observer with a given characteristic quasipolynomial and an exponentially stable observer. The behavior of the estimation errors of the observers is described by a linear homogeneous autonomous system of neutral type. Moreover, for the first type of observer, it is possible to specify a desired characteristic quasipolynomial of the system describing the error before its design (i.e., to set in advance the rate of convergence of the observer's estimate to the solution of the original system). In the case of the second type of observer, the system describing the estimation error of the solution is exponentially stable. Note that it is generally impossible to control the coefficients of the characteristic equation. However, the exponential stability of the system describing the estimation error ensures the convergence of the estimate to the solution at an exponential rate.

The methods for constructing controllers and observers developed in this study involve standard operations with polynomials and polynomial matrices and are easily implemented in modern computer algebra packages.

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APPENDIX

Proof of Theorem 1.

We introduce the neutral system

$$\dot{x}(t) = A'(p_D, \lambda_h)x(t) + c'(\lambda_h)u(t), \quad t > 0, \tag{A.1}$$

and define the controller

$$u(t) = \beta_1(p_D)x_1(t), \quad \dot{x}_1(t) = \mathcal{K}_1[x(t)] + \mathcal{K}_2[x_1(t)], \quad t > 0, \tag{A.2}$$

where $x_1 \in \mathbb{R}$ is an auxiliary variable, $\mathcal{K}_1 \in \mathfrak{Q}^{1\times n}$, $\mathcal{K}_2 \in \mathfrak{Q}$, and $\beta_1(p) \in \mathbb{R}_0[p]$.

For any given quasipolynomial (13) there exists a controller (A.2) such that $|W_1(p,\lambda)| = g(p,\lambda)$ if and only if conditions (19) are valid [15]. Here, $W_1(p, e^{-ph})$ is the characteristic matrix of the closed-loop system (A.1), (A.2):

$$W_1(p,\lambda) = \begin{bmatrix} pI_n - A'(p,\lambda) & -\beta_1(p)c'(\lambda) \\ -K_1(p,\lambda) & p - K_2(p,\lambda) \end{bmatrix},$$

with $K_i(p,\lambda) = \mathcal{K}_i[e^{pt}]e^{-pt}\Big|_{e^{-ph}}$. Letting

$$\beta_0(p) = \beta_1(p), \quad \mathcal{L}_1 = \mathcal{K}_1', \quad \mathcal{L}_2 = \mathcal{K}_2 \tag{A.3}$$

in equations (9) gives $\big(W_1(p,\lambda)\big)' = W_z(p,\lambda)$. Therefore, $|W_z(p,\lambda)| = g(p,\lambda)$, which implies the existence of an observer (9) with the desired characteristic quasipolynomial.

Proof of Theorem 2. For system (A.1) there exists a controller (A.2) making the closed-loop system exponentially stable if and only if conditions (20) are valid [15]. Therefore, repeating the proof of Theorem 1 with the necessary modifications, we show the existence of an exponentially stable observer (9). The proof of this theorem is complete.

Proof of Theorem 3.

Necessity. Conditions (25) [15] are necessary and sufficient for modal controllability in the class of feedback controllers based on measurements of the state vector x. Therefore, conditions (25) are necessary for modal controllability in the class of feedback controllers based on measurements of the observed output (2).

Let us prove the necessity of conditions (19). Suppose that system (1), (2) is modally controllable in the class of controllers (21). By assumption, the controller (21) ensures equality (24) for some given polynomial (23). Consider system (A.1) and define a controller of the form

$$\begin{gathered} u(t) = -\alpha_1(p_D)x_1(t), \\ \dot{x}_1(t) = \mathcal{Q}'_{33}[x_1(t)] + \mathcal{Q}'_{23}[x_2(t)], \\ \dot{x}_2(t) = \alpha_1(p_D)c'(\lambda_h)x_1(t) + A'(p_D, \lambda_h)x_2(t) + \mathcal{Q}'_{12}[x_3(t)], \\ \dot{x}_3(t) = \alpha_0(p_D)b'(\lambda_h)(x(t) + x_2(t)) + \mathcal{Q}'_{11}[x_3(t)], \quad t > 0. \end{gathered} \tag{A.4}$$

Let $\widehat{W}(p, e^{-ph})$ be the characteristic matrix of system (A.1),(A.4). Obviously, the matrix $\left(\overline{W}(p,\lambda)\right)'$ is obtained from the matrix $\widehat{W}(p,\lambda)$ by permuting the rows and columns of blocks with numbers 2 and 4. Therefore, we write $E_{24}\widehat{W}(p,\lambda)E_{24}^{-1} = \left(\overline{W}(p,\lambda)\right)'$, where the matrix E_{24} swaps the rows of suitable-size blocks with numbers 2 and 4 when multiplied by any matrix on the left. Hence, $|\widehat{W}(p,\lambda)| = \chi(p,\lambda)$, meaning that system (A.1) is modally controllable in the sense of [15] (i.e., by a feedback controller based on the state function x), and conditions (19) express a modal controllability criterion for system (A.1). The necessity of conditions (19) and (25) is established.

Sufficiency. Consider a given polynomial (23). We prove the sufficiency part by providing a design scheme for a controller (21) ensuring equality (24).

1. We define a state-feedback controller of the form

$$u(t) = \alpha_0(p_D)x_1(t), \quad \dot{x}_1(t) = \mathcal{Q}_{12}[x(t)] + \mathcal{Q}_{11}[x_1(t)], \quad t > 0. \tag{A.5}$$

The notation in (A.5) is the same as in (21). Due to conditions (25), for any polynomial $\chi_1(p,\lambda)$ (23) there exists [9] a controller (A.5) such that the characteristic matrix $W_x(p, e^{-ph})$ of the closed-loop system (1), (A.5),

$$W_x(p,\lambda) = \begin{bmatrix} pI_n - A(p,\lambda) & -b(\lambda)\alpha_0(p) \\ -Q_{12}(p,\lambda) & p - Q_{11}(p,\lambda) \end{bmatrix}, \tag{A.6}$$

satisfies the equality

$$|W_x(p,\lambda)| = \chi_1(p,\lambda). \tag{A.7}$$

Thus, the controller (A.5) has been constructed.

2. Under condition (19) (see Theorem 1), for any given polynomial $\chi_2(p,\lambda)$ (23) there exists an observer (9) with a given characteristic quasipolynomial such that the characteristic matrix (12) satisfies the relation

$$|W_z(p,\lambda)| = \chi_2(p,\lambda). \tag{A.8}$$

Thus, the observer (9) has been constructed.

3. Using the parameters of the controller (A.5) and observer (9), we write the controller (21) with the additional assignment

$$\mathcal{Q}_{23} = \mathcal{L}_1, \quad \mathcal{Q}_{33} = \mathcal{L}_2. \tag{A.9}$$

Let us show equality (24) for the characteristic matrix $\overline{W}(p, e^{-ph})$ of the closed-loop system (1), (2), (21). For this purpose, we introduce the matrix

$$\Gamma = \begin{bmatrix} I_n & 0_{n\times 1} & 0_{n\times n} & 0_{n\times 1} \\ 0_{1\times n} & 1 & 0_{1\times n} & 0 \\ -I_n & 0_{n\times 1} & I_n & 0_{n\times 1} \\ 0_{1\times n} & 0 & 0_{1\times n} & 1 \end{bmatrix}. \tag{A.10}$$

Direct verification yields

$$\Gamma\overline{W}(p,\lambda)\Gamma^{-1} = \widetilde{W}(p,\lambda), \tag{A.11}$$

$$\widetilde{W}(p,\lambda) = \begin{bmatrix} pI_n - A(p,\lambda) & -b(\lambda)\alpha_0(p) & 0_{n\times n} & 0_{n\times 1} \\ -Q_{12}(p,\lambda) & p - Q_{11}(p,\lambda) & -Q_{12}(p,\lambda) & 0_{1\times 1} \\ 0_{n\times 1} & 0 & pI_n - A(p,\lambda) & -Q_{23}(p,\lambda) \\ 0_{n\times 1} & 0 & -\alpha_1(p)c'(\lambda) & p - Q_{33}(p,\lambda) \end{bmatrix}.$$

From equalities (A.6), (A.7), (A.8), (A.11), and (12) it follows that $\overline{W}(p,\lambda) = \Gamma\overline{W}(p,\lambda)\Gamma^{-1} = \chi(p,\lambda)$. The proof of Theorem 3 is complete.

Proof of Theorem 4. The idea of proving Theorem 4 is quite similar to that of proving Theorem 3, so we will present only its brief scheme.

Necessity. 1. Suppose that system (1), (2) closed by the controller (21) is exponentially stable. We form the sets Δ_0 and Δ_1 from Remark 2 (see (16)). If the first condition in (26) is violated, then for any $\varepsilon_0 < 0$ there exists $p_0 \in \mathbb{C}$, $\mathbf{Re}\, p_0 \geqslant \varepsilon_0$, such that $\operatorname{rank}\big[W(p_0, e^{-p_0 h}), b(e^{-p_o h})\big] < n$. In this case, for any controller of the form (21), the number p_0 remains in the spectrum of the closed-loop system (1), (2), (21), i.e., $p_0 \in \Delta_0$. Therefore, condition (17) fails and, consequently, system (1), (2), (21) cannot be exponentially stable.

If the second condition in (26) is violated, then there exists $\lambda_0 \in \mathbb{C}$, $|\lambda_0| \leqslant 1$, such that $\operatorname{rank}\big[D(\lambda_0)\big] < n$. Obviously, for any controller of the form (21), the closed-loop system (1), (2), (21) satisfies $\lambda_0 \in \Delta_1$. Hence, condition (18) is violated. The necessity of conditions (26) is established.

2. Now we prove the necessity of conditions (20). Consider system (A.1) closed by the controller (A.4). Assuming sequentially that the first or second conditions in (20) are violated, similar to (1), we show that the closed-loop system (A.1), (A.4) cannot be exponentially stable.

Sufficiency. We describe a design scheme for the controller (21) and then prove the exponential stability of the closed-loop system.

1. Following [9], we construct the controller (A.5) exponentially stabilizing the closed-loop system (1), (A.5). Conditions (26) ensure the possibility of constructing such a controller. In this case, the characteristic matrix of the closed-loop system (1), (A.5) has the form (A.6).

2. We construct the exponentially stable observer (9). Conditions (20) ensure the possibility of constructing such an observer. In this case, the characteristic matrix of the homogeneous system (9) has the form (12).

3. Using the parameters of the controller (A.5) and observer (9), we write the controller (21) with the matrices assigned by (A.9).

Let us show the exponential stability of system (1), (21). For this purpose, we apply the following nondegenerate transformation of the variables:

$$\mathrm{col}[x, x_1, x_2, x_3] = \Gamma^{-1}\mathrm{col}[\tilde{x}, \tilde{x}_1, \tilde{x}_2, \tilde{x}_3],$$

with the matrix Γ given by (A.10). This transformation yields a new system with the characteristic matrix $\widetilde{W}(p, e^{-ph})$, where the matrix $\widetilde{W}(p, \lambda)$ has the form (A.11). The resulting system will be called the system $\widetilde{\Sigma}$.

Due to the representation of the matrix $\widetilde{W}(p, \lambda)$, the components $\tilde{x}_2$ and $\tilde{x}_3$ are determined by a separate system (a subsystem of the system $\widetilde{\Sigma}$) whose characteristic matrix coincides with (12). Therefore, the system determining the components $\tilde{x}_2$ and $\tilde{x}_3$ is exponentially stable. In other words, there exist positive constants γ_1 and γ_2 such that

$$\|\tilde{x}_i(t)\| \leqslant \gamma_1 e^{-\gamma_2 t}, \quad t > 0, \quad i = 2, 3. \tag{A.12}$$

Consider the system corresponding to the first two rows of the blocks of the matrix $\widetilde{W}(p, \lambda)$. Since the components $\tilde{x}_2$ and $\tilde{x}_3$ are determined separately, they can be treated as an inhomogeneous part in the system under consideration. Then the components $\tilde{x}$ and $\tilde{x}_1$ satisfy the inhomogeneous system for which the characteristic matrix of the corresponding homogeneous system coincides with (A.6). Hence, the above homogeneous system is exponentially stable and, in view of (A.12), there exist positive constants γ_3 and γ_4 such that

$$\|\tilde{x}(t)\| \leqslant \gamma_3 e^{-\gamma_4 t}, \quad \|\tilde{x}_i(t)\| \leqslant \gamma_3 e^{-\gamma_4 t}, \quad t > 0, \quad i = \overline{1, 3}. \tag{A.13}$$

These inequalities imply the exponential stability of the system $\widetilde{\Sigma}$ and, consequently, of system (1), (21). The proof of Theorem 4 is complete.

REFERENCES

1. Dolgii, Yu.F. and Surkov, P.G., *Matematicheskie modeli dinamicheskikh sistem s zapazdyvaniem* (Mathematical Models of Dynamic Systems with Delay), Yekaterinburg: Ural University, 2012. https://elar.urfu.ru/bitstream/10995/45629/1/978-5-7996-0772-2_2012.pdf (Accessed March 17, 2025.)

2. Kolmanovskiy, V.B. and Nosov, V.R., Systems with an After-effect of the Neutral Type, *Autom. Remote Control*, 1984, vol. 45, no. 1, pp. 1–28.

3. Poloskov, I.E., *Metody analiza sistem s zapazdyvaniem* (Analysis Methods for Systems with Delay), Perm: Perm State National Research University, 2020. http://www.psu.ru/files/docs/science/books/mono/poloskov-metody-analiza-sistem.pdf

4. Khartovskii, V.E., *Upravlenie lineinymi sistemami neitral'nogo tipa: kachestvennyi analiz i realizatsiya obratnykh svyazei* (Control of Linear Systems of Neutral Type: Qualitative Analysis and Feedback Implementation), Grodno: Grodno State University, 2022.

5. Grebenshchikov, B.G., Asymptotic Properties and Stabilization of a Neutral Type System with Constant Delay, *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes*, 2021, vol. 17, no. 1, pp. 81–96. https://doi.org/10.21638/11701/spbu10.2021.108

6. Bulgakov, B.V., *Kolebaniya* (Oscillations), Moscow: Izd-vo Tekhniko-teoreticheskoi Lit-ry, 1954.

7. Krasovskii, N.N. and Osipov, Yu.S., On the Stabilization of Motions of a Plant with Delay in the Control System, *Izv. Akad. Nauk SSSR. Tekh. Kibern.*, 1963, no. 6, pp. 3–15.

8. Osipov, Yu.S., On the Stabilization of Controlled Systems with Delay, *Differ. Uravn.*, 1965, vol. 1, no. 5, pp. 606–618.

9. Pandolfi, L., Stabilization of Neutral Functional-Differential Equations, *J. Optim. Theory Appl.*, 1976, vol. 20, no. 2, pp. 191–204. https://doi.org/10.1007/BF01767451

10. Lu, W.S., Lee, E., and Zak, S., On the Stabilization of Linear Neutral Delay-Difference Systems, *IEEE Transact. Autom. Control*, 1986, vol. 31, no. 1, pp. 65–67. https://doi.org/10.1109/TAC.1986.1104115

11. Rabah, R., Sklyar, G.M., and Barkhayev, P.Y., Stability and Stabilizability of Mixed Retarded-Neutral Type Systems, *ESAIM Control, Optimization and Calculus of Variations*, 2012, vol. 18, no. 3, pp. 656–692. https://doi.org/10.1051/cocv/2011166

12. Dolgii, Yu.F. and Sesekin, A.N., Regularization Analysis of a Degenerate Problem of Impulsive Stabilization for a System with Time Delay, *Tr. Inst. Mat. Mekh. UrO RAN*, 2022, vol. 28, no. 1, pp. 74–95. https://doi.org/10.21538/0134-4889-2022-28-1-74-95

13. Hu, G.D. and Hu, R., A Frequency-Domain Method for Stabilization of Linear Neutral Delay Systems, *Syst. Control Lett.*, 2023, vol. 181, art. no. 105650. https://doi.org/10.1016/j.sysconle.2023.105650

14. Hale, J.K. and Verduyn Lunel, S.M., Strong Stabilization of Neutral Functional Differential Equations, *IMA J. Math. Control Inf.*, 2002, vol. 19, no. 1–2, pp. 5–23. https://doi.org/10.1093/imamci/19.1_and_2.5

15. Metel'skii, A.V., Spectrum Assignment for a System of Neutral Type, *Diff. Equat.*, 2024, vol. 60, no. 1, pp. 101–126. https://doi.org/10.1134/S0012266124010099

16. Minyaev, S.I. and Fursov, A.S., Topological Approach to the Simultaneous Stabilization of Plants with Delay, *Diff. Equat.*, 2013, vol. 49, no. 11, pp. 1423–1431. https://doi.org/10.1134/S0012266113110098

17. Watanabe, K., Finite Spectrum Assignment and Observer for Multivariable Systems with Commensurate Delays, *IEEE Trans. Autom. Control*, 1986, vol. AC–31, no. 6, pp. 543–550. https://doi.org/10.1109/TAC.1986.1104336

18. Wang, Q.G., Lee, T.H., and Tan, K.K., *Finite Spectrum Assignment Controllers for Time Delay Systems*, Springer-Verlag, 1999. https://doi.org/10.1007/978-1-84628-531-8

19. Metel'skii, A.V., Spectral Reduction, Complete Damping, and Stabilization of a Delay System by a Single Controller, *Diff. Equat.*, 2013, vol. 49, no. 11, pp. 1405–1422. https://doi.org/10.1134/S0012266113110086

20. Marchenko, V.M., Control of Systems with Aftereffect in Scales of Linear Controllers with Respect to the Type of Feedback, *Diff. Equat.*, 2011, vol. 47, no. 7, pp. 1014–1028. https://doi.org/10.1134/S0012266111070111

21. Metel'skii, A.V. and Khartovskii, V.E., Criteria for Modal Controllability of Linear Systems of Neutral Type, *Diff. Equat.*, 2016, vol. 52, no. 11, pp. 1453–1468. https://doi.org/10.1134/S0012266116110070

22. Khartovskii, V.E., Modal Controllability for Systems of Neutral Type in Classes of Differential-Difference Controllers, *Autom. Remote Control*, 2017, vol. 78, no. 11, pp. 1941–1954. https://doi.org/10.1134/S0005117917110017

23. Fridman, E., *Introduction to Time-Delay Systems: Analysis and Control*, Birkhäuser, 2014. https://doi.org/10.1007/978-3-319-09393-2

24. Furtat, I. and Fridman, E., Delayed Disturbance Attenuation via Measurement Noise Estimation, *IEEE Transaction on Automatic Control*, 2021, vol. 66, no. 11, pp. 5546–5553. https://doi.org/10.1109/TAC.2021.3054238

25. Karpuk, V.V. and Metel'skii, A.V., Complete Calming and Stabilization of Linear Autonomous Systems with Delay, *J. Comput. Syst. Sci. Int.*, 2009, vol. 48, no. 6, pp. 863–872. https://doi.org/10.1134/S1064230709060033

26. Metel'skii, A.V., Khartovskii, V.E., and Urban, O.I., Solution Damping Controllers for Linear Systems of the Neutral Type, *Diff. Equat.*, 2016, vol. 52, no. 3, pp. 386–399. https://doi.org/10.1134/S0012266116030125

27. Metel'skii, A.V., Complete and Finite-Time Stabilization of a Delay Differential System by Incomplete Output Feedback, *Diff. Equat.*, 2019, vol. 55, no. 12, pp. 1611–1629. https://doi.org/10.1134/S0012266119120085

28. Khartovskii, V.E., Finite Stabilization and Finite Spectrum Assignment by a Single Controller Based on Incomplete Measurements for Linear Systems of the Neutral Type, *Diff. Equat.*, 2024, vol. 60, no. 5, pp. 655–676. https://doi.org/10.1134/S0012266124050094

29. Khartovskii, V.I. and Urban, O.I., Incomplete Measurements-Based Finite Stabilization of Neutral Systems by Controllers with Lumped Commensurate Delays, *Autom. Remote Control*, 2025, vol. 86, no. 1, pp. 1–19. https://doi.org/10.31857/S000523102501

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